Appendix A

Recursive method for solving the block Tridiagonal Matrix Equation

To solve the equation (3.32)

$$A(N)\delta y(N-1)+B(N)\delta y(N)+C(N)\delta y(N+1)=F(N)\ ,\ 2\leq N\leq L-1 \qquad \text{A1}$$

for

$$\delta y(N)=[\delta p(N),\delta n(N),\delta\psi(N)]^T\ ,\qquad 2\leq N\leq L-1$$

assuming that equation A1 is transformed into the following equation involving unknown vectors at two points instead of three:

$$B'(N)\delta y(N)+C'(N)\delta y(N+1)=F'(N) \qquad \text{A2}$$

this is written explicitly for $\delta y(N)$ to yield

$$\delta y(N)=B'(N)^{-1}F'(N)-B'(N)^{-1}C'(N)\delta y(N+1) \qquad \text{A3}$$

Division point N is decreased by one, and result is substituted in equation A1to yield

$$\left[B(N)-A(N)B'(N-1)^{-1}C'(N-1)\right]\delta y(N)+C(N)\delta y(N+1)=F(N)-A(N)B'(N-1)^{-1}F'(N-1) \qquad \text{A4}$$

comparison A4 with A2 given the recursive relations

$$\begin{cases} B'(N)=B(N)-A(N)B'(N-1)^{-1}C'(N-1) \\ C'(N)=C(N) \\ F'(N)=F(N)-A(N)B'(N-1)^{-1}F'(N-1) \qquad 3\leq N\leq L-1 \end{cases} \qquad \text{A5}$$

hence for N=2, direct comparison of equations A2and A1 yields $\delta y(1)=0$

$$B'(2)=B(2),\ C'(2)=C(2),\ F'(2)=F(2). \qquad \text{A6}$$

Thus, starting with equation A6, $B'(N),C'(N),F'(N)$ in equation A5 are determined for N=3, 4, …, L-1 where the inverse matrix for is directly obtained through the Sylvester's formula. Namely, let

$B'=(b'_{ij})$ and $B^{-1}=(\beta'_{ij})$ with i,j=1,2,3.

then the interrelation between these two matrices is given by,

$$\begin{cases} \beta_{11}=\dfrac{b'_{22}b'_{33}-b'_{32}b'_{23}}{\det B'} \quad \beta_{12}=\dfrac{b'_{32}b'_{13}-b'_{12}b'_{33}}{\det B'} \quad \beta_{13}=\dfrac{b'_{12}b'_{13}-b'_{12}b'_{22}}{\det B'} \\ \beta_{21}=\dfrac{b'_{31}b'_{23}-b'_{21}b'_{33}}{\det B'} \quad \beta_{22}=\dfrac{b'_{11}b'_{33}-b'_{31}b'_{13}}{\det B'} \quad \beta_{23}=\dfrac{b'_{21}b'_{13}-b'_{11}b'_{33}}{\det B'} \\ \beta_{31}=\dfrac{b'_{21}b'_{32}-b'_{31}b'_{32}}{\det B'} \quad \beta_{32}=\dfrac{b'_{31}b'_{12}-b'_{11}b'_{32}}{\det B'} \quad \beta_{33}=\dfrac{b'_{11}b'_{22}-b'_{21}b'_{12}}{\det B'} \end{cases} \qquad \text{A7}$$

The final step is to calculate $\delta y(N)=L-1, L-2, \ldots .2$ by equation A.3, where $\delta y(L)=0$ is used as the starting condition.